

# A Foundation Theory of Quantum Mechanics

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## Abstract

The nRules are empirical regularities that were discovered in macroscopic situations where the outcome is known. When they are projected theoretically into the microscopic domain they predict a novel ontology including the frequent collapse of an atomic wave function, thereby defining an nRule based foundation theory. Future experiments can potentially discriminate between this and other foundation theories of (non-relativistic) quantum mechanics. Important features of the nRules are: (1) they introduce probability through probability current rather than the Born rule, (2) they are valid independent of size (micro or macroscopic), (3) they apply to individual trials, not just ensembles of trials. (4) they allow all observers to be continuously included in the system without ambiguity, (5) they account for the collapse of the wave function without introducing new or using old physical constants, and (6) in dense environments they provide a high frequency of stochastic localizations of quantum mechanical objects. Key words: measurement, stochastic choice, state reduction.

## Introduction

The nRules are four auxiliary rules that guide or direct the application of Schrödinger's equation. They are verbal instructions that say how the Schrödinger equation should be applied to any quantum mechanical system. One of them (nRule 4) modifies the Hamiltonian in a way that will be shown in the next section.

Every equation in physics is accompanied by verbal instructions of some sort. Otherwise it would not be physics - it would be mathematics. The nRules

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are different from most auxiliary rules in that they impose limitations on the Schrödinger equation that have no classical counterpart. They are more assertive and have direct dynamical consequences.

The Born rule is an auxiliary rule of standard quantum mechanics; however, it is not necessary as a governing tenet. It can be derived from other rules, and the notion of probability can be introduced into quantum mechanics through *probability current*. This has been done in two cases: The *nRules* [1, 2] and the *oRules* [3, 4]. All other quantum mechanical auxiliary rules that do not use probability current in this way will be referred to as *sRules*. This paper is primarily concerned with the *nRules*.

There are several considerations that go into the formulation of the *nRules*. It is required that wave collapses or state reductions are ‘objective’ and ‘self-generated’. That is, the rules describe how a wave function can collapse automatically inside a closed system, independent of any outside observer or measuring device. In addition, the rules are intended from the beginning to apply to individual interactions (single trials) not just ensembles of trials, and to be independent of size. It is initially decided to use probability current to introduce probability. And finally, the *nRules* are discovered by testing their validity in well-known macroscopic situations where they can be shown to be empirically correct. They are then projected theoretically into the microscopic domain.

As a consequence of these requirements it is found that the *nRules* allow the *primary* observer to be continuously included in the system. This is similar to classical physics in that an observer who investigates an external system has the option of extending the system to include himself. The *sRules* do not always let that happen. In the Copenhagen case the Born rule requires that the primary observer remain outside. He is allowed to peek at the system from time to time to determine the Born connection at that moment, but he is not allowed to stay in the system. Other *sRules* such as the many world thesis of Everett and the GRW/CSL reduction theory of GianCarlo Ghirardi and his associates also allow the observer to be in the system in the classical sense [5, 6, 7].

Another consequence is that any and all secondary observers can be included in the system in a continuous and unambiguous way. This removes the paradoxical results that are associated with the Schrödinger cat experiment, and with all other ambiguities that result when a secondary observer is admitted into the system. Therefore, *all* conscious observers have a place in quantum mechanical systems under the *nRules*.

The above comments also apply to the *oRules* with the exception of the “observer independence” of state reduction, inasmuch as the *oRules* require that

a conscious observer must be present for a measurement to occur – following von Neumann’s suggestion to that effect. An objection to the nRules is considered in the Conclusion together with a discussion of the experimental possibilities.

## The nRules

We define *ready components* to be the basis components of state reduction. These are the components that are chosen to survive the collapse of the wave function. They are underlined throughout the paper. Components that are not ready are called *realized components* and appear without an underline.

The first nRule describes how ready components are introduced into solutions of Schrödingers equation.

**nRule (1):** *If an irreversible interaction produces a complete component that is discontinuous with its predecessor in some variable, then it is a ready component. Otherwise a component is realized.*

[**note:** A *complete component* is one that includes all the (anti)symmetrized objects in the universe. Each included object is itself complete in that it is not a partial expansion in some representation.]

The second rule establishes the existence of a stochastic trigger. The flow per unit time of square modulus is given by the square modular current  $J$ , and the total square modulus of the system is given by  $s$ .

**nRule (2):** *A systemic stochastic trigger strikes a ready component with a probability per unit time equal to the positive probability current  $J/s$  flowing into it. A realized component is not stochastically chosen.*

[**note:** The division of  $J$  by  $s$  automatically normalizes the system at each moment of time. Currents rather than functions are normalized under these rules.]

The collapse of a wave is given by nRule (3)

**nRule (3):** *When a ready component is stochastically chosen it will become a realized component, and all other (non-chosen) components will go immediately to zero.*

[**note:** We can choose to amend nRule (3) so that other components do *not* go to zero. It does no harm to let them stand unchanged after a stochastic hit because there will be no further consequence. Square modulus has no physical meaning in the nRules, and current no longer flows into or out of these components because the Hamiltonian has passed them by – as will be explained in the next section. The spent components would have the status of a “phantom” as defined

in Ref. 1. The decision to let these not-chosen components go to zero or to let them stand is like the decision in standard quantum mechanics to renormalize (or not) after a measurement]

The fourth nRule has a less obvious meaning.

**nRule (4):** *A ready component cannot transmit probability current to other components or advance its own evolution.*

[**note:** The fourth nRule is enforced by withholding a ready component’s Hamiltonian as explained in the next section.]

## The Quantum Algorithm

To understand nRule (4) and the note under nRule (3), we go back to the Hamiltonian formalism and adopt a modification that I call the “Quantum Algorithm”. Instructions for the application of the classical Hamiltonian are: *Beginning with the initial boundary conditions of a closed system, the Hamiltonian drives all of the system’s particles and all of its interactions **into the indefinite future**.* This defines a classical unitary evolution that goes on forever.

The quantum algorithm modifies these instructions to read:

**QUANTUM ALGORITHM:** *Beginning with the initial boundary conditions of a closed system, the Hamiltonian drives all of the system’s particles and all of its interactions **up to but not beyond** the next ready component(s).* This introduces a non-unitary process at each ready component that locates sites (or possible sites) of quantum measurement. It also locates other non-unitary evolutions that are assumed to occur microscopically.

The fourth nRule is implicit in this algorithm. In fact, the quantum algorithm can be used in place of nRule (4).

It works like this.

The horizontal line labeled  $S_0$  in Fig. 1 represents the initial state of a system that evolves continuously in time. It is a single (complete) component driven by the Hamiltonian  $H_0 + H_{01}$  until it encounters and includes the discontinuous irreversible gap  $G_{01}$ . The interaction Hamiltonian  $H_{01}$  will drive probability current across this gap to the first ‘ready’ component in the figure (i.e., the first shaded area), but current will not pass beyond that point. As a result, the shaded component in  $G_{01}$  will accumulate square modulus. This blocking of probability current (required by nRule 4) is accomplished by truncating the initial Hamiltonian so it does not contain the term  $H_1$ .

A stochastic hit on the ready component in  $G_{01}$  will change it to a ‘realized’ component according to nRules (3), thereby launching a new solution  $S_1$  of

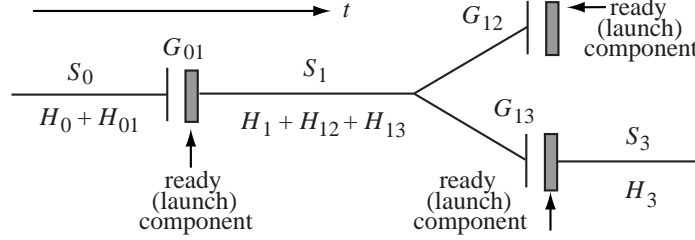


Figure 1: Collapse  $S_0$  to  $S_1$  then to  $S_3$

Schrödinger's equation. It will also make the initial system state irrelevant or go to zero (i.e., it *reduces*  $S_0$  to  $S_1$ ). The ready component is also called a *launch component*, for it contains all of the *initial conditions* of the launched solution  $S_1$ . These initial conditions are continuously updated by current flow into the launch component prior to the stochastic hit. We therefore say that the collapse of the wave across  $G_{01}$  is characterized by a *new solution that is brought about by a new Hamiltonian*, where the initial conditions are contained in the launch component. All of this can just as well be accomplished if, in place of nRule (4), the quantum algorithm is directly applied; for in that case, the second Hamiltonian  $H_1 + H_{12} + H_{13}$  will substitute for  $H_0 + H_{01}$  the moment initial conditions contained in  $G_{01}$  are launched to give the new solution  $S_1$ . The variables of  $H_0$  and  $H_1$  are indexed to the variables of  $S_0$  and  $S_1$  respectively.

Once  $S_1$  is launched, the new Hamiltonian will carry the system up to and across the next discontinuous and irreversible gap(s). Let this consist of the two parallel gaps  $G_{12}$  and  $G_{13}$  as shown in Fig. 1, so the launch components in  $G_{12}$  and  $G_{13}$  present the stochastic chooser with two possibilities. The Hamiltonian driving  $S_1$  is  $H_1 + H_{12} + H_{13}$  as stated above, where  $H_{12}$  and  $H_{13}$  are discontinuous interaction terms, so positive current will flow into both of the parallel ready components. However, only one will be chosen. Suppose it is  $S_3$  together with Hamiltonian  $H_3$  with everything else going to zero or just being ignored. In Fig. 1 we conclude with this solution and its Hamiltonian. As in  $G_{01}$ , the flow of this system at both  $G_{12}$  and  $G_{13}$  is non-unitary.

## A Particle Capture

These nRules were initially discovered by examining many different quantum mechanical interactions that involve classical instruments like detectors or counters. They successfully describe all the cases considered; and on this basis, they

are claimed to describe *any* observable quantum phenomena. This section is the first example of this macroscopic kind, where an elementary particle is captured by a detector. There can be no question about the macroscopic events that occur as a result of that interaction. The nRules describe ‘what happens’ in this case without any attempt to give a theoretical explanation, or to justify those rules beyond the fact that they work.

We apply Schrödinger’s equation to a particle interacting with a detector in order to demonstrate the first stage of Fig. 1. The interaction beginning at time  $t_0$  is given by

$$\Phi(t \geq t_0) = \psi(t)d_0(t) + \underline{d}_1(t) \quad \text{applying } H_0 + H_{01} \quad (1)$$

where the second component is zero at  $t_0$  and increases in time. The free particle  $\psi(t)$  here interacts with the ground state detector  $d_0(t)$  and is driven by the Hamiltonian  $H_0$ . The interaction Hamiltonian  $H_{01}$  produces a probability current flow from the first component to the second component in Eq. 1, where the latter is the detector in its capture state. The gap between these two components is discontinuous because the particle is completely outside the detector in the first component, and it is completely inside the detector in the second component; and the two are not bridged by intermediate components that are continuous in particle variables. This interaction is also irreversible. Therefore, the gap given by the + sign satisfies nRule (1), making  $\underline{d}_1(t)$  a ‘ready’ component as indicated by the underline. Each component in Eq. 1 is multiplied by the associated total environment (not shown), assuring detector decoherence in this case and satisfying the requirement that each component is complete.

Current flowing into  $\underline{d}_1(t)$  will increase its square modulus and update its content through the interaction Hamiltonian  $H_{01}$ , but any further evolution is blocked by nRule (4). This makes  $d_1(t)$  the launch component that establishes and updates the initial conditions of the next solution of the Schrödinger equation. Since positive probability current flows into this ready component, it is subject to a stochastic hit as specified by nRule (2). If that happens at a time  $t_{sc}$ , then nRule (3) will require a state reduction giving

$$\Phi(t \geq t_{sc} > t_0) = d_1(t) \quad \text{now applying } H_1 \quad (2)$$

At  $t_{sc}$  the Hamiltonian  $H_1$  is applied to the realized detector component  $d_1(t)$ , allowing it to evolve on its own.

The time dependence of  $d_1(t)$  in Eq. 2 refers to the evolution of the detector after capture. The time dependence of  $\underline{d}_1(t)$  in Eq. 1 refers to changes in the launch component due to current flow from the first component. The latter

changes include the increase in the square modulus of the launch, plus information that continuously updates the boundary conditions that are contained in the launch. The distinction between these two changes will be better clarified in next section. The necessity of nRule (4) will become more apparent in subsequent sections where it is shown that a macroscopic body (such as a counter) cannot be otherwise described by Schrödinger's equation in this non-Born rule protocol.

## Free Neutron Decay

When the nRules are applied to microscopic systems they become ‘speculations’ rather than empirically knowable regularities. This section is the first example of that kind. The nRules are here projected into a realm in which the resulting ontology is very different from the description given by standard quantum mechanics. Again, there is no attempt to justify the nRules beyond the fact that they work well on the macroscopic level. Our speculation amounts to requiring that there is no fundamental distinction between the macro and the microscopic, so that the same rules – in particular the nRules – apply in both domains.

A *free neutron decay* is given by

$$\Phi(t \geq t_0) = n(t) + \underline{ep}\overline{\nu}(t)$$

where the second component is zero at  $t_0$  and increases in time. It is shown as a package of three particles that are the boundary conditions of the neutron's decay. It's a ready component, although it not necessary to underline the entire component – one state will do. To satisfy the requirement of completion, each component is multiplied by the total environment (not shown) even though the neutron and its decay products are an isolated system.

This case provides a good example of how the launch component  $\underline{ep}\overline{\nu}(t)$  is a function of time beyond its increase in square modulus. Assume that the neutron moves across the laboratory in a wave packet of finite width. At each moment the launch component will ride *with the packet*, having its same shape and group velocity. This component contains the boundary conditions of the next solution of the Schrödinger equation – the solution that appears when  $\underline{ep}\overline{\nu}(t)$  is stochastically chosen at  $t_{sc}$ . The launch component is time dependent because it increases in square modulus *and* because it follows the motion of the neutron. This is how it updates the boundary conditions of the decay. However, nRule (4) insures that the launch will not evolve dynamically beyond itself before becoming a realized component at the time of stochastic choice. Only then will

the neutron disappear by decaying into separate particles  $e(t)p(t)\bar{\nu}(t)$  that spread out on their own, still correlated in conserved quantities.

As a consequence of nRules (3) and (4), the decay occurs at only one stochastically chosen point along the path; whereas, standard quantum mechanics results in a superposition of decays that are spread out all along the path. The nRules apply to individual locations along the path, and the sRules apply to ensembles of those locations. This shows that the nRules are more definite than the sRules, although they remain less deterministic than classical physics.

Specific values of the electron's momentum are not stochastically chosen by this reduction. All the possible values of momentum are included in  $e(t)p(t)\bar{\nu}(t)$  after the transients associated with  $\Delta E \Delta t$  have died out, where  $\Delta t$  begins only after the new Hamiltonian is applied. For the electron's momentum to be determined in a specific direction away from the decay site, a detector in that direction must be activated. That will require another nRule equation involving a stochastic hit on the detector.

Experimentally it should be possible in a single trial to find the momentum of the electron, the proton, and the antineutrino with sufficient accuracy to locate the decay somewhere along the original path of the neutron with an accuracy consistent with Heisenberg. However, this information cannot be used to disqualify the nRule or the standard theoretical description of what happened. Experiment cannot confirm or deny the existence of intermediate boundaries (or wave reductions) that fix the locus the decay before the separate particles have been detected. In this and many other cases, our theoretical projection of the nRule gives us a microscopic ontology that is unique, but it is experimentally indistinguishable from that of standard quantum mechanics.

## A Series of Discontinuities

The section on "Particle Capture" allowed the nRules to be given their first macroscopic formulation. The next macroscopic step is to consider a series of discontinuities as represented by a particle counter  $A$  that is activated by a nearby radioactive source.

Consider the series of components  $A_0, A_1, A_2, A_3, \dots$  that are serially connected to each other by discontinuous and irreversible gaps. In standard quantum mechanics, a series like this is given by

$$\Phi(t \geq t_0) = A_0(t) + A_1(t) + A_2(t) + A_3(t) + \dots \quad (3)$$

where only  $A_0$  is non-zero at time  $t_0$ . The other components gain amplitude by



virtue of probability current flowing from  $A_0$  to  $A_1$ , then to  $A_2$ , and then to  $A_3$ , etc. They are a succession of counter readings whose subscripts record the number of particles that have been captured from a nearby radioactive source. Let  $A_0$  mean that no particles have been captured,  $A_1$  means that one particle has been captured, etc. We do not include the intermediate particle field in Eq. 3. It simplifies the example to imagine that the apparatus states interact directly with each other.

Applying the *first three* nRules to this case, the envelope of the amplitudes of the components in Eq. 3 will form a pulse that moves from left to right. All the current receiving components will be ready components according to nRule (1), so Eq. 3 will take the form

$$\Phi(t \geq t_0) = A_0(t) + \underline{A}_1(t) + \underline{A}_2(t) + \underline{A}_3(t) + \dots \quad (4)$$

But there is a problem. In the absence of nRule (4), the component  $\underline{A}_2(t)$  will acquire some degree of amplitude the moment  $\underline{A}_1(t)$  acquires amplitude. Probability current might then flow simultaneously into  $\underline{A}_1(t)$  and  $\underline{A}_2(t)$ , in which case there might be a stochastic hit on  $\underline{A}_2(t)$  before there is a hit on  $\underline{A}_1(t)$ . That is a very unphysical result for a macroscopic counter. The fourth nRule is added to insure that *this does not occur*.

When nRule (4) is applied, Eq. 4 becomes

$$\Phi(t \geq t_0) = A_0(t) + \underline{A}_1(t) \quad (5)$$

and this guarantees that the state  $\underline{A}_2(t)$  will not be stochastically chosen before  $\underline{A}_1(t)$ . The effect of nRule (4) is therefore to guarantee that  $\underline{A}_1(t)$  is not passed over, and this alone is an indispensable requirement in this non-Born protocol. It is a macroscopic necessity that alone justifies our adopting the truncated Hamiltonian of the quantum algorithm.

Probability current flowing from  $A_0$  to  $\underline{A}_1$  in Eq. 5 will result in a stochastic hit on  $\underline{A}_1$  at some time  $t_{sc1}$ . When that happens we get the first particle capture

$$\Phi(t \geq t_{sc1} > t_0) = A_1(t) + \underline{A}_2(t) \quad (6)$$

where  $\underline{A}_2(t)$  is zero at  $t_{sc1}$  and increases in time. Following this, another stochastic hit at  $t_{sc2}$  gives the second particle capture

$$\Phi(t \geq t_{sc2} > t_{sc1} > t_0) = A_2(t) + \underline{A}_3(t) \quad (7)$$

and so fourth. In Eqs. 5, 6, and 7, the correct sequential order of counter states is guaranteed by nRule (4).

It is characteristic of the sRules (i.e., any Born-based theory) that there is only one solution (Eq. 3) to the Schrödinger equation for the given initial conditions, whereas the nRules provide a separate solution for each discontinuous gap (Eqs. 5, 6, 7, etc.). The launch component in each case will contain all of the updated boundary conditions of the next solution that become effective the moment it is stochastically chosen. So  $\underline{A}_1$  in Eq. 5 is the launch component into the new solution in Eq. 6 and contains all of the boundary conditions that apply at time  $t_{sc1}$ . The realized component  $A_1(t_{sc1})$  in Eq. 6 is the initial boundary of that solution.

There is no contradiction between the predictions of the nRules and the standard sRules. The nRules are concerned with the probability in an individual trial that the next stochastic hit will occur in the next interval  $dt$  of time. Opposed to this, the sRules are concerned with the distribution of an *ensemble* of states at some finite time  $T$  after the apparatus is turned on. These different rules-sets ask different questions having different answers. However, either one of these protocols can be successfully mapped onto the same counter, so there can be no observational contradiction.

It is no strain to see that Eq. 5 applies to *microscopic states* as well, for serial order is just as important in these cases. Atomic states that decay from an initial excited state  $A_0$  will go to the next lower energy state  $A_1$  and then lower to  $A_2$  without skipping a step – unless that possibility is allowed by the Hamiltonian. If it is not allowed, then  $A_1$  will not be skipped over. As in the macroscopic case, nRule (4) is an essential moderator of any serial sequence at the atomic level. Otherwise, the second order component  $A_2$  might be stochastically chosen before  $A_1$  is chosen, and that would be unphysical. Although the nRules are empirically discovered by investigating macroscopic systems, they can be extended to this microscopic system, thereby supporting the claim that the rules are independent of size.

## Parallel Discontinuities

Macroscopic parallel branching is also used to check the correctness and generality of the nRules. The equation of state involving a laboratory apparatus  $A$

is given by

$$\Phi(t \geq t_0) = A_0(t) + \underline{A}_r(t) + \underline{A}_l(t) \quad (8)$$

where  $\underline{A}_r(t)$  and  $\underline{A}_l(t)$  are the eigenvectors of stochastic choice that are initially equal to zero and increase in time. Each one receives probability current from the first component that makes it a candidate of a state reducing stochastic hit – like  $G_{12}$  and  $G_{13}$  in Fig. 1. Each is a launch component, where  $\underline{A}_r(t)$  contains the boundary conditions of a launch toward the right in Fig. 2, and  $\underline{A}_l(t)$  contains the boundary conditions of a launch toward the left. The dashed lines in Fig. 2 are the initially forbidden transitions.

Think of these components as representing two macroscopic particle counters, where  $A_0$  means that neither one has yet made a capture,  $A_r(t)$  means that the one on the right is the first to make a capture,  $A_l(t)$  means that the one on the left is the first to make a capture, and the final state  $A_f(t)$  represents the system when each counter has made a single capture. Let each counter turn off after a single capture. Again, we simplify by not including the particle fields. If the launch component  $\underline{A}_r(t)$  is stochastically chosen in Eq. 8 at time  $t_{scr}$ , the resulting state reduction will yield

$$\Phi(t \geq t_{scr} > t_0) = A_r(t) + \underline{A}_f(t)$$

where  $\underline{A}_f(t)$  is the launch component into final state of the system. When it is stochastically chosen at time  $t_{scf}$  the system will be in its final state  $\Phi(t \geq t_f > t_{scr} > t_0) = A_f(t)$ . This sequence will go in a counterclockwise direction if the launch component  $\underline{A}_l(t)$  in Eq. 8 is stochastically chosen. As in the series case, all of these launch components are time dependent because their square moduli increase in time *and* because at each moment they take on the updated boundary conditions that apply to the new solution in case of a stochastic hit at that moment.

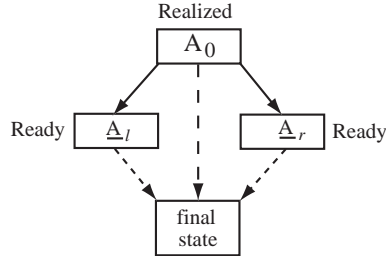


Figure 2: Possible parallel decay routes

The fourth nRule therefore has the effect of forcing these macroscopic counters into either a clockwise or a counterclockwise path in the classical sense. Without nRule (4), a second order transition might skip over the intermediate components to score a direct stochastic hit on  $A_f$  without one of the intermediate component being definitely involved. This is unphysical behavior for a macroscopic system. So nRule (4) transforms the initial superposition of Eq. 8 into two classical alternatives because it does not allow intermediate components such as  $\underline{A}_l$  or  $\underline{A}_r$  to be skipped over. Here again we see the indispensability of nRule (4) if macroscopic objects are to be quantum mechanically described with a non-Born protocol.

The same will be true of *microscopic* parallel systems. An “irreversible discontinuity” imposes an abrupt and lasting change of a distinctive kind in some part of the universe – even in a microscopic case. For instance, let Fig. 2 represent two alternative routes from a high-energy atomic state  $A_0$  to the ground state  $A_f$ . The two photons that are released along each path will leave an irreversible record that will be different for each path (assuming non-degeneracy); so if the two photons associated with the clockwise path are found in the wider universe, then the clockwise path must have been stochastically chosen. It is not possible for *all four* photons to be found in a single trial. It will be either the two photons from the left or the two from the right. The released photons are the abrupt and lasting change referred to above, and the distinctive characteristics of the photons along each path removes the possibility of interference between the paths. Statistically, the two paths are a mixture; so in any individual trial, only one eigenstate  $\underline{A}_r$  or  $\underline{A}_l$  would be traversed.

More generally of any microscopic or macroscopic series/parallel combination of paths, any single path segment that follows and precedes an irreversible discontinuous gap will be phase independent of all the other such path segments in the combination, and will be correctly described by the nRules.

## Add an Observer

When an observer is added to the system it is necessarily macroscopic. The results of this section are empirically undeniable, and may therefore be considered a further check on the correctness of the nRules.

First, imagine that an observer is present to witness the detector capture in Eq. 1 as it would appear following any one of the sRules of standard quantum

mechanics. The resulting equation would be

$$\Phi(t \geq t_0) = \psi d_0 B_0 + d_{1w} B_0 \rightarrow d_{1d} B_1 \quad (9)$$

where only the first component is non-zero at  $t_0$  and decreases in time. To simplify, the time dependence of the states in these components is not explicitly shown. The state  $d_{1w}$  represents the detector when the captured particle has advanced to ‘just inside’ the *window*. This component will evolve continuously from that point on, carrying the influence of the capture from the window end of the detector to the *display* end, which is given by the display state  $d_{1d}$ . The arrow in Eq. 9 represents this continuous evolution. So  $d_{1w} B_0 \rightarrow d_{1d} B_1$  is a single component of Schrödinger’s equation that evolves in time, representing the continuous/classical change that occurs inside the detector after a capture.

The state  $B_0$  in Eq. 9 is the brain state of the observer who witnesses the detector in its ground state. The brain will continue in that state after capture until the signal has moved through the detector to the display. At the display end it becomes  $B_1$ , which is the brain state of the observer who witnesses the capture. The change from  $B_0$  to  $B_1$  is therefore continuous and largely classical. The quantum mechanical discontinuity is confined to the particle’s jump when it goes from being outside of the detector to being just inside the window. Of course the detector also jumps discontinuously from  $d_0$  to  $d_{1w}$  at the same time.

The trouble with Eq. 9 is that it is a potential cat-like disaster. If the interaction time between  $d_0$  and the particle field is long enough (i.e., if the incoming particle is spread out sufficiently in space), the initial signal will travel through to the display before the first component has gone to zero. This means that two very different brain states  $B_0$  and  $B_1$  will appear simultaneously in that equation, and that will result in an ambiguity reminiscent of Schrödinger’s cat experiment.

When the nRules are applied to this case, Eq. 9 is modified to

$$\Phi(t \geq t_0) = \psi d_0 B_0 + \underline{d}_{1w} B_0 \quad (10)$$

where  $\underline{d}_{1w} B_0$  is the launch component of the gap that contains the boundary conditions of the next solution. This component is equal to zero at  $t_0$  and increases in time. Again, it is sufficient to underline only one state in the launch component to indicate that the entire component is ‘ready’. Following a stochastic hit on this component at time  $t_{sc}$ , we get the next solution

$$\Phi(t \geq t_{sc} > t_0) = d_{1w} B_0 \rightarrow d_{1d} B_1 \quad (11)$$

where the resulting realized component is  $d_{1w} B_0$  at  $t_{sc}$  and evolves continuously in time to become  $d_{1d} B_1$ .

There is no ambiguity here because there is only one brain state in Eq. 10, and there is only one brain state ‘at a time’ in Eq. 11. The Copenhagen rules give us one equation (Eq. 9) in which there is a potential cat-like ambiguity, whereas the nRules give us two equations (Eqs. 10 and 11) in which there is no ambiguity. Equation 10 applies *before* a stochastic hit, and Eq. 11 applies *after* a stochastic hit. Here again, the standard sRules compress two solutions having two separate boundary conditions into a single equation with only the initial conditions  $d_0 B_0$ , so these solutions are *not properly grounded* in all of the boundary conditions that apply. On the other hand, the nRules give two solutions, each based on different boundary conditions given by  $d_0 B_0$  and  $d_{1w} B_0$ .

In standard sRule theories, the Born rule provides the probability connection between theory and observation through the square modulus. The nRules establish this connection differently. An observer’s brain is placed in the system in contact with the apparatus (e.g., a detector or counter), and his experience is predicted by the effect of that interaction on the brain. The probability of this happening during a given time interval  $dt$  is determined by the probability current. As in classical physics, the primary observer can imagine that his own brain is part of the system. This means that the primary and secondary observers have the same ontological status under the nRules, and that both have the same ‘reality’, as any other object in the universe. The primary observer is not banished to an Olympian mountaintop where he studies the distant universe as something apart from himself.

## Multiple Parallel Sequences

Construct a network of macroscopic counters and sources that allow an initial state  $AB_0$  to decay to either eigencomponents  $AB_1$  or  $AB_2$  or  $AB_3$ , where  $A$  is an apparatus that is witnessed by a brain state  $B$ . The subscript on  $B$  denotes the state of the apparatus that is seen by the brain. Ignore the window/display distinction, and again assume that the components are time dependent. After the first stochastic choice that carries the system from the first to the second row of Fig. 3, there is a secondary choice that carries the system to the third row.

There are six possible sequences in this diagram. In the many-world universe of Everett these branches all run together in a single superposition. The observer who inhabits one of these branches cannot be aware of his own alter-ego in another branch, for that would disqualify the idea. Everett showed that once begun, one of these sequences will proceed without any further involvement

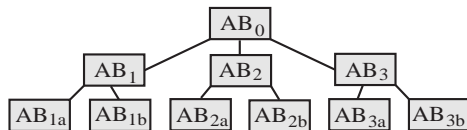


Figure 3: Six possible sequences

with any other sequence. That means that the observer on one branch of this system will not be aware of his alter-ego on another branch. The branches (or sequences) therefore proceed independent of one another, even though they are regarded (by Everett) as a single superposition. As before, the particle field is not included in the analysis.

The nRules tell a different story. They do not run all these solutions together. The nRules say that each stochastic choice in Fig. 3 is the occasion of a collapse of the wave and the launch of a new solution of Schrödinger's equation. Each of the six possible sequences consists of two collapses that follow the initial state  $AB_0$ . There will therefore be three separate equations that carry the initial state into a final state. For the sequence  $AB_0, AB_1, AB_{1b}$ , those equations are

$$\begin{aligned}
 \Phi(t \geq t_0) &= AB_0 + \underline{AB}_1 + \underline{AB}_2 + \underline{AB}_3 \\
 \Phi(t \geq t_{sc1} > t_0) &= AB_1 + \underline{AB}_{1a} + \underline{AB}_{1b} \\
 \Phi(t \geq t_{sc2} > t_{sc1} > t_0) &= AB_{1b}
 \end{aligned}$$

where in each case the launch components are initially zero. Evidently the system is not a superposition of all the possible sequences; but rather, each sequence is a series of individual decays that proceed independent of other sequences.

Although this construction is illustrated with observable macroscopic instruments, it would work as well with a microscopic array of atomic states where there can be no observers. This again is because every irreversible & discontinuous gap and accompanying stochastic hit leaves a mark on the wider universe that indelibly records the choice. In the microscopic case this mark will take the form of an emitted photon or other irreversible happening recorded in the 'memory' of the universe, much as the memory of each alter-ego is irreversibly affected in Everett's theory.

## Atomic Absorption and Emission

Applying this scheme to the case of atomic absorption and emission, the atom in its ground state interacts with a laser field  $\gamma_n(t)$  containing  $n$  photons of the

excitation frequency. These are photons of frequency 0-1, where 0 refers to the ground state  $a_0(t)$ , and 1 refers to the excited state  $a_1(t)$ . The nRules then give

$$\Phi(t \geq t_0) = \gamma_n a_0 \Leftrightarrow \gamma_{n-1} a_1 + \gamma_{n-1} \underline{a}_0 \otimes \gamma \quad (12)$$

where only the first component is zero at time  $t_0$ . Again, only one state in the ready component  $\gamma_{n-1} \underline{a}_0 \otimes \gamma$  needs to be underlined, and it is understood that every state is a function of time. The double arrow ( $\Leftrightarrow$ ) represents a reversible Rabi oscillation associated with the laser that begins at  $t_0$ . When the atom is in the excited state  $a_1$  a spontaneous emission to ground becomes a possibility, represented here by the ready component. When that component is stochastically chosen the atom goes to ground, emitting a photon  $\gamma$  that came to it from the laser beam.

If the atom begins in the excited state and is exposed to a laser beam, we get

$$\Phi(t \geq t_0) = \gamma_n a_1 \Leftrightarrow \gamma_{n+1} a_0 + \gamma_n \underline{a}_0 \otimes \gamma \quad (13)$$

where again, only the first component is non-zero at  $t_0$ . Again, a stimulated emission oscillation begins immediately, where a spontaneous emission from the excited state is represented by the ready component. Except for the fact that Eq. 13 has one more photon than Eq. 12, the two equations are identical. It cannot matter if the oscillation begins in  $a_0$  or in  $a_1$ .

## A Laser

Given a four level atom with a ground state  $a_0$  and three excited states  $a_1$ ,  $a_2$ ,  $a_3$  of increasing energy. It is immersed in a laser field of  $n$  photons  $\gamma$  with an energy that connects levels  $a_1$  and  $a_2$ . The atom is initially pumped into the short-lived state  $a_3$  and is quickly dropped into  $a_2$  by an irreversible energy loss involving some dissipative process that may be molecular collisions, or possibly the spontaneous emission of a 3-2 photon.

$$\Phi(t \geq t_0) = \gamma_n a_3 + \gamma_n \underline{a}_2 \otimes e_x \quad (14)$$

where the second component is zero at  $t_0$  and increases in time. The symbol  $e_x$  represents other parts of the system not appearing in the first component (like adjacent molecules or radiation fields) that absorb the energy difference. With a stochastic hit on the ready component in this equation at time  $t_{sc1}$ , the system becomes

$$\begin{aligned} \Phi(t \geq t_{sc1} > t_0) = & \gamma_n a_2 \otimes e_x \Leftrightarrow \gamma_{n+1} a_1 \otimes e_x + \gamma_n \underline{a}_1 \otimes e_x \otimes \gamma \text{ (metastable)} \\ & + \gamma_{n+1} \underline{a}_0 \otimes e_x \otimes e_{xx} \end{aligned}$$



where only the first component is non-zero at  $t_{sc1}$ . The metastable decay in the first row is a long-lived spontaneous photon emission coming off the first component. The symbol  $e_{xx}$  in the short-lived decay component (second row) represents that part of the environment that takes up the energy difference between  $a_1$  (in the second component) and  $a_0$ . The short-lived decay product is more likely to be stochastically chosen than the metastable one, so after a second hit at time  $t_{sc2}$  we have preferentially

$$\Phi(t \geq t_{sc2} > t_{sc1} > t_0) = \gamma_{n+1}a_0 \otimes e_x \otimes e_{xx}$$

Comparing the original state  $\gamma_n a_3$  with the final state  $\gamma_{n+1}a_0 \otimes e_x \otimes e_{xx}$ , it is clear that the energy difference between  $a_3$  and  $a_0$  is the energy of the new photon in the laser beam plus the two dissipative processes  $e_x$  and  $e_{xx}$ . The cycle is repeated many times resulting in pumping many new photons into the laser beam. Evidently each photon pumped into the beam requires two stochastic hits – i.e., two wave collapses associated with two non-unitary processes. I do not call these “measurements” because I think it is best to reserve that word for non-unitary processes that involve macroscopic instruments.

## Localization

Localization is essential if macroscopic objects are to have the location properties that correspond to our common experience with them. This property is not contained in the Schrödinger equation by itself, for objects subject only to that equation will expand forever due to their uncertainty in momentum. Therefore, localization must be provided for by auxiliary rules of some kind. The Copenhagen sRules use a macroscopic instrument to locate a quantum mechanical particle, but this will not apply when macroscopic encounters do not play a fundamental role. There is also the ‘bootstrap’ question of how a macroscopic instrument can itself be located, considering that it (and all other such instruments) began their existence as a collection of free hydrogen and helium atoms following recombination about 14 billion years ago. The Schrödinger equation cannot itself localize (i.e., collapse) a collection of this kind. On the other hand, the nRules make no essential use macroscopic objects. These rules are shown below to provide a purely microscopic localization of matter in certain matter-dense environments.

Let a photon raise an atom to an excited state, after which the atom drops down again by spontaneously emitting a photon. The incoming photon is assumed to be spread out widely over space. The atom is also spread out over

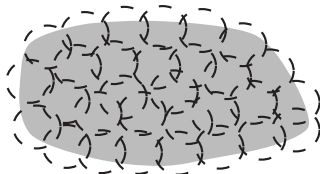


Figure 4: Scattered wavelets

space by an amount that exceeds its *minimum volume*. This is defined to be the smallest volume that the atom can occupy consistent with its initially given uncertainty of momentum. The atom in Fig. 4 (shaded area) is assumed to be spread far beyond this volume prior to its interaction with the photon. As the incoming photon passes over the enlarged atom, we assume that the scattered radiation will appear as a superposition of many photons that originate from different parts of the atom's extended volume as shown in Fig. 4 – these are the small wavelets in the figure. The correlations between the nucleus of the atom and the orbiting electrons must be preserved, even though the atomic superposition is spread out over a much larger volume. That is, the smaller dimensions of the minimal volume atom must be unchanged during its expansion, so the potential energy of the orbiting electrons is unchanged. The atom could not otherwise act as the center of a ‘characteristic’ photon emission. This means that the incident photon will engage the compact atom throughout every part of the enlarged volume.

However, if the atom is in a rich and random environment its volume in Fig. 4 will not remain coherent. Imagine that external influences break it up into  $n$  *bubbles* of lesser volume (but no smaller than the minimum volume) that are decoherent relative to one another. We will then have

$$\Phi(r, t_{de} \geq t > t_0) = a(r, t) \rightarrow \Sigma_n a_n(r, t)$$

by the continuous decoherent process described in Ref. 1 where  $t_{de}$  is a time when decoherence is complete. In this equation the initially coherent (shaded) atomic state  $a(r, t)$  becomes  $n$  separate decoherent bubbles, where each is represented by  $a_n(r, t)$ . This summation is not an expansion representation of the atom. Rather, it is a mixture of decoherent components, each of which is ‘complete’ because the atom is entirely contained in each bubble. When the incoming photon  $\gamma$  is included the above equation is

$$\Phi(r, t_{de} \geq t > t_0) = \gamma(t)a(r, t) \rightarrow \gamma(t)\Sigma_n a_n(r, t)$$

Let the photon interact with each of the bubbles after time  $t_{de}$ . Each will

then give rise to a Rabi oscillation between the ground state  $a_n(r, t)$  and the excited state given by  $\hat{a}_n(r, t)$ .

$$\Phi(r, t \geq t_{de} > t_0) = \Sigma_n[\gamma(t)a_n(r, t) \Leftrightarrow \hat{a}_n(r, t) + \underline{a}_n(r, t) \otimes \gamma_n(t)]$$

where  $\gamma_n$  is the photonic wavelet (in Fig. 4) associated with the  $n^{th}$  bubble. There is a chance that the  $k^{th}$  ready state in the square bracket will be stochastically chosen at a time  $t_{sc}$ . In that case

$$\Phi(r, t \geq t_{sc} > t_{de} > t_0) = a_k(r, t) \otimes \gamma_k(r, t)$$

When this happens the *ground state atom is reduced* to the size of the  $k^{th}$  bubble. This illustrates one of the ways that the world around us becomes localized. Solar photons and greenhouse photons in the Earth's atmosphere will produce widespread scattering events of this kind that localize the atmosphere, and hence the correlated surface of the earth. The same will be true of the scattering of the sun's light as it enters the waters of the Earth's lakes and oceans. In addition, there are many examples of irreversible reactions resulting from discontinuous quantum jumps in the rich biosphere of the earth – all contributing to our experience of being well defined in space. A localization of this kind is discussed in another paper [8].

The above reduction is not possible under standard sRules because it does not involve a macroscopic object. An exception would seem to be the GRW/CSL theory because in this case a reduction to one of the bubbles is possible. However, it is very unlikely.

## Other Applications

The nRules have been satisfactorily applied to a number of different macroscopic cases. For instance, the Born rule can be 'derived' when an observer looks at the terminal results in a typical physics experiment. In this case an interaction in an initially normalized system is allowed to go to completion. If there is more than one resulting launch eigencomponent, it follows from the nRules that one of them will be chosen with a probability equal to the square modulus of that component – which is the time integrated probability current into that component.

The nRules also give good results when we investigate how an initially independent observer engages a particle detector *during* its interaction with a particle (Ref. 1). Also in Ref. 1, we look at the case of two observers, where one joins the other during an observed interaction between a particle and a detector.

In a separate paper the Schrödinger cat experiment is examined in all of its variations [9]. In one version the cat is initially conscious and is made unconscious by a mechanical device that is initiated by a radioactive emission. In another version the cat is initially unconscious and is made conscious by an alarm clock that is set off by a radioactive emission. In still another version, the cat is awakened by a natural internal alarm (such as hunger) that is in competition with an external mechanical alarm. In all these cases, the nRules are shown to accurately and unambiguously predict the expected experience of the cat at any moment of time. And finally, an external observer is assumed to open the box containing the cat at any time during any one of these experiments; and when that happens, his experience of the cat's condition is correctly predicted by the nRules.

In all the above macroscopic cases, plus the five examined in this paper, the nRules are found to be entirely correct. Furthermore, no macroscopic case has been found in which they are not correct. It is easy to believe that rules that appear to be so generally true at the macroscopic level are also true at the microscopic level; and when applied microscopically, the nRule description of events is qualitatively different from that given by any of the sRules. Several microscopic applications have been described in this paper: neutron decay, series and parallel microscopic discontinuities, microscopic multiple sequences, atomic absorption and emission, lasing, and localization. Others are investigated in Ref. 1 where we look at spin states, decoherence, and Rabi oscillations, neither of which brings about a state reduction under the nRules.

There is an unexpected bonus contained in the nRules. In another paper [10], nRule (4) insures the forward flow of probability current. In thermodynamics the forward direction is only very probable; but the nRules *require* that time's arrow flows in the right way. Since nRule (1) contains the word "predecessor", it must be reworded to say that the side of the gap with the higher entropy identifies the ready component. With that change, it is clear that nRule (4) will prevent current from arriving at the gap from the "wrong" direction, because that would require current flow within the ready component.

## Conclusion

One objection to these nRules is their verbal form. To be taken seriously it is said that a theory of physics must be formulated mathematically. For instance, nRule (2) requires the existence of a systemic stochastic trigger. The GRW/CSL theory has a similar requirement calling for the existence of a white noise or

randomly fluctuating field  $\omega(x, t)$ . There are some functional differences between the two, like the selectivity of the trigger that only affects ready components, as opposed to the noise that affects everything. But the main objection to the trigger is that it is not represented by a mathematical function that can be included in the equation of motion as can  $\omega(x, t)$ . This, I believe, is an aesthetic objection that might or might not be correct. We've come to think of physical theory as being only mathematically expressible, but that might be too limited a view. Verbal rules, auxiliary to a dynamical principle, are no less precise, or true, because of their form.

A preference for the nRules is also an aesthetic choice at this point, for there is no experiment that can now distinguish it from other foundation theories. That may happen, but for the time being we are left with a three-step methodology that takes us from the empirical to the theoretical and back to experimental possibilities.

1. The nRules are auxiliary rules of the non-relativistic Schrödinger equation. They are empirically correct in all the macroscopic situations that have been investigated. I believe these investigations are sufficiently complete to claim that the nRules are generally valid empirical laws or formulas - like Balmer's spectral series, or Planck's blackbody radiation law.
2. It is easy to imagine that wide-spread regularities that appear in one domain will also appear in another domain. If that is true in this case, then the nRules will apply in microscopic systems as well as in macroscopic systems. We assume here that the fundamentals are the same in both domains, so both large and small things follow the same rules - the nRules.
3. The question is: Does experimental evidence exist that favors this theory compared with other current theories of quantum mechanics and quantum measurement? At present the answer is "no". However, there are prospects. The GRW/CSL theory predicts the existence of a physical constant  $\lambda$  that governs the rate at which a particle is affected by the stochastic noise (Ref. 7). This is supposedly a very small number whose existence has not yet been experimentally confirmed. If it is found at some future time, then the microscopic generality of the nRules will no longer be defensible. On the other hand, if experiments do not confirm the existence of the GRW/CSL physical constant then all rival theories will remain viable candidates, including the microscopic nRules. I know of no experiment that can decisively verify the microscopic nRules, but I am hopeful that one will be found.

The more immediate virtue of the nRules is their heuristic value when thinking about microscopic processes. The rules are no help when calculating prob-

abilities, but they do provide a skeletal outline that connects a microscopic network of stochastic choices, making the options more transparent and the ontology better defined. So in addition to the other desirable properties enumerated in this paper (as in the abstract), the nRules help one understand why the Schrödinger equation does what it does microscopically. Of course they are not just a heuristic device. They claim to be an account of what really happens to atomic systems.

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